# On Angels and Demons: Strategic (De)Construction of Dynamic Models

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# Games and Logics for Graphs Modifications

# Sabotage Games

**Sabotage games** are two-player games played on a directed graph $\mathcal{G}$.
Starting from an initial node $v_1$:

- **The Demon** erases **one** edge $e$ anywhere in $\mathcal{G}$.
- **The Traveler** then moves to an adjacent node $v_2$ (if possible).

This process generates a finite $\pi = v_1, \dots, v_n$

The Demon wins if the last element $v_n$ of $\pi$ is not in a specified set $\mathcal{P}$.

Determining whether there exists a Demon strategy $\mathfrak{S}$ such that all compatible plays are winning for the Demon is a **PSPACE-complete** problem.

# Obstruction Games

**Obstruction games** are two-player games played on directed, *weighted, serial* graphs $\mathcal{M}$.

From an initial node $v_1$ with maximum allowed cost $n$:

- **The Demon** selects a strict subset $E$ of edges incident to $v_1$, with total cost at most $n$.
- **The Traveler** chooses a node $v_2$ with $(v_1, v_2) \notin E$.

After restoring $E$, play continues from $v_2$, producing an infinite sequence $\pi$.

The Demon wins iff $\pi$ satisfies a given temporal property $\psi$.

# Obstruction Logic

**Obstruction Logic (OL)** extends Temporal Logic with a Strategic Modality $\langle \dagger^n \rangle \psi$

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle \dagger^n \rangle \psi$$

$$\psi := \mathsf{X}\, \varphi \mid \varphi\, \mathsf{U}\, \varphi \mid \varphi\, \mathsf{R}\, \varphi$$

$\langle \dagger^n \rangle \psi$ means there is Demonic strategy for selecting edges such that each path compatible with the strategy satisfies $\psi$.

**The model checking problem for OL is PTIME-complete**

# Two Perspectives on Graph Modification

| **SG** | **OL** |
| --- | --- |
| global deletion | local obstruction |
| persistent changes | temporary changes |
| modal reasoning | strategic temporal reasoning |
| PSPACE | PTIME |

Can we combine strategic temporal reasoning with global graph modification?

# Generalized Games for Graph Modification

Let $\mathfrak{M} = (S, \rightarrow, \mathcal{V}, C)$ be a directed, labeled, graph, in which $C : S \times S \rightarrow \mathbb{N}^{+}$. Given an initial state $s$ and two natural numbers $n, m$:

- **The Demon** deletes a subset $E$ of edges anywhere in $\mathfrak{M}$ whose sum is at most $n$;
- **The Angel** adds a new subset of edges $E'$ whose sum is at most $m$;
- **The Traveler** chose a node $s'$ s.t. $(s, s') \in (\rightarrow \setminus E) \cup E'$

By repeating rounds, we get an *infinite* play $\pi$. The coalition formed by the Demon and the Angel wins iff $\pi$ satisfies a temporal property $\psi$.

We obtain:

- infinite **Sabotage Games** if the Angel does nothing at each turn
- A **new kind of games** if the Demon does nothing at each turn.
- **Obstruction Games** if the Angel adds at each turn $i+1$ the edges that were deleted by The Demon at turn $i$, and does nothing otherwise.

We developed a strategic logic for each of these variants of the game.

# Reasoning About Infinite Sabotage Games: Strategic Deconstruction Logic

# Semantics I

A *model* $\mathfrak{M} = (S, \to, \mathcal{V}, C)$ is a directed labeled serial graph in which a cost is associated to any pair of states

A $n$-play is an infinite sequence of pointed models

$$\pi = (\mathfrak{M}_0, s_0), (\mathfrak{M}_1, s_1), \ldots$$

where $\mathfrak{M}_{i+1}$ is obtained from $\mathfrak{M}_i$ by deleting a set of edges whose cost is at most $n$ and $s_i \xrightarrow{\mathfrak{M}_{i+1}} s_{i+1}$

A **n-Demonic Strategy** is a Function $\mathfrak{S}$ associating to each pointed model $(\mathfrak{M}, s)$ a set $A \subseteq \xrightarrow{\mathfrak{M}}$ whose cost is at most $n$.

# Semantics II

$$\varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle \pentagram^n \rangle \psi \qquad \psi := \mathsf{X}\,\varphi \mid \varphi\,\mathsf{U}\,\varphi \mid \varphi\,\mathsf{R}\,\varphi$$

$(\mathfrak{M}, s) \models \langle \pentagram^n \rangle \psi$ iff there is an $n$-strategy $\mathfrak{S}$ s.t. for all $\pi \in Out(\mathfrak{S}, (\mathfrak{M}, s))$ we have $\pi \models \psi$

# Some Results About SDL

**CTL is a syntactic fragment of SDL.**

**There are SDL formulae that cannot be expressed by OL.**

**The Model-Checking Problem for SDL is PSPACE-complete.**

- Lower Bound = reduction from QBF
- Upper Bound = APTIME Algorithm

So much work to do and so little time to do it.

# Open Problems

It remains unknown whether there exist OL formulae that cannot be expressed in SDL.

We have only considered memoryless strategies so far. What about strategies with memory? Is this concept even meaningful in SDL?

What about classical logical problems such as axiomatizability and satisfiability?

# Some References

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